Jury’s Stability test

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INTRUDUCTION:

- For continuous-time systems, the Routh–Hurwitz criterion offers a simple and convenient technique for determining the stability of low-ordered systems.
- since the stability boundary in the z-plane is different from that in the s-plane, the Routh–Hurwitz criterion cannot be directly applied to discrete-time systems if the system characteristic equation is expressed as a function of z.
- A stability criterion for discrete-time systems that is similar to the Routh–Hurwitz criterion and can be applied to the characteristic equation written as a function of z

So ; jury test is used directly on characteristic equation which is in Z-transform .

JURY procedures :

First we have jury’s table :

TABLE 7-2 Array for Jury’s Stability Test

z^0	z^1	z^2	$\dots$	z^{n-k}	$\dots$	z^{n-1}	z^n
a_0	a_1	a_2	$\dots$	a_{n-k}	$\dots$	a_{n-1}	a_n
a_n	a_{n-1}	a_{n-2}	$\dots$	a_k	$\dots$	a_1	a_0
b_0	b_1	b_2	$\dots$	b_{n-k}	$\dots$	b_{n-1}	
b_{n-1}	b_{n-2}	b_{n-3}	$\dots$	b_{k-1}	$\dots$	b_0	
c_0	c_1	c_2	$\dots$	c_{n-k}	$\dots$		
c_{n-2}	c_{n-3}	c_{n-4}	$\dots$	c_{k-2}	$\dots$		
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$			
l_0	l_1	l_2	l_3				
l_3	l_2	l_1	l_0				
m_0	m_1	m_2					

The jury's tests:

$$Q(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0 = 0, \qquad a_n > 0 \tag{7-13}$$

Then form the array as shown in Table 7-2. Note that the elements of each of the even-numbered rows are the elements of the preceding row in reverse order. The elements of the odd-numbered rows are defined as

$$\begin{aligned} b_k &= \begin{vmatrix} a_0 & a_{n-k} \\ a_n & a_k \end{vmatrix}, \qquad c_k = \begin{vmatrix} b_0 & b_{n-1-k} \\ b_{n-1} & b_k \end{vmatrix} \\ d_k &= \begin{vmatrix} c_0 & c_{n-2-k} \\ c_{n-2} & c_k \end{vmatrix} \cdots \end{aligned} \tag{7-14}$$

The necessary and sufficient conditions for the polynomial $Q(z)$ to have no roots outside or on the unit circle, with $a_n > 0$, are as follows:

$$\begin{aligned} Q(1) &> 0 \\ (-1)^n Q(-1) &> 0 \\ |a_0| &< a_n \\ |b_0| &> |b_{n-1}| \\ |c_0| &> |c_{n-2}| \\ |d_0| &> |d_{n-3}| \\ &\vdots \end{aligned} \tag{7-15}$$

Note that for a second-order system, the array contains only one row. For each additional order, two additional rows are added to the array. Note also that for an nth-order system, there are a total of $n + 1$ constraints.

Jury's test may be applied in the following manner:

1. Check the three conditions $Q(1) > 0$, $(-1)^n Q(-1) > 0$, and $|a_0| < a_n$, which requires no calculations. Stop if any of these conditions are not satisfied.
2. Construct the array, checking the conditions of (7-15) as each row is calculated. Stop if any condition is not satisfied.

EXAMPLE:

Consider again the system of Example 6.4 (and Example 7.3). Suppose that a gain factor K is added to the plant, and it is desired to determine the range of K for which the system is stable. Now, from Example 6.4, the system characteristic equation is

$$1 + KG(z) = 1 + \frac{(0.368z + 0.264)K}{z^2 - 1.368z + 0.368} = 0$$

or

$$z^2 + (0.368K - 1.368)z + (0.368 + 0.264K) = 0$$

The Jury array is

z^0	z^1	z^2
$0.368 + 0.264K$	$0.368K - 1.368$	1

The constraint $Q(1) > 0$ yields

$$1 + (0.368K - 1.368) + (0.368 + 0.264K) = 0.632K > 0 \Rightarrow K > 0$$

The constraint $(-1)^2 Q(-1) > 0$ yields

$$1 - 0.368K + 1.368 + 0.368 + 0.264K > 0 \Rightarrow K < \frac{2.736}{0.104} = 26.3$$

The constraint $|a_0| < a_2$ yields

$$0.368 + 0.264K < 1 \Rightarrow K < \frac{0.632}{0.264} = 2.39$$

Thus the system is stable for

$$0 < K < 2.39$$

The system is marginally stable for $K = 2.39$. For this value of K, the characteristic equation is

$$z^2 + (0.368K - 1.368)z + (0.368 + 0.264K)|_{K=2.39} = z^2 - 0.488z + 1 = 0$$

The roots of this equation are

$$z = 0.244 \pm j0.970 = 1\angle(\pm 75.9°) = 1\angle(\pm 1.32 \text{ rad}) = 1\angle(\pm \omega T)$$

EXAMPLE:

Suppose that the characteristic equation for a closed-loop discrete-time system is given by the expression

$$Q(z) = z^3 - 1.8z^2 + 1.05z - 0.20 = 0$$

The first conditions of Jury's test are

$$\begin{aligned} Q(1) &= 1 - 1.8 + 1.05 - 0.2 = 0.05 > 0 \\ (-1)^3 Q(-1) &= -[-1 - 1.8 - 1.05 - 0.2] = 4.05 > 0 \\ |a_0| &= 0.2 < a_3 = 1 \end{aligned}$$

The Jury array is calculated to be

z^0	z^1	z^2	z^3
−0.2	1.05	−1.8	1
1	−1.8	1.05	−0.2
−0.96	1.59	−0.69	

where the last row has been calculated as follows:

$$b_0 = \begin{vmatrix} -0.2 & 1 \\ 1 & -0.2 \end{vmatrix} = -0.96, \qquad b_1 = \begin{vmatrix} -0.2 & -1.8 \\ 1 & 1.05 \end{vmatrix} = 1.59$$

$$b_2 = \begin{vmatrix} -0.2 & 1.05 \\ 1 & -1.8 \end{vmatrix} = -0.69$$

Hence the last condition is

$$|b_0| = 0.96 > |b_2| = 0.69$$